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Performance analysis of physical-layer network coding with QPSK modulation in wireless IoT networks

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Abstract

In this paper, we mathematically analyze bit-error rate (BER) and diversity order of physical-layer network coding (PNC) scheme in the two-way relay channel (TWRC) under the assumption of multiple antennas at relay node. We assume the quadrature phase shift keying (QPSK) modulation for simple data transmission in IoT applications, and max–min transmit antenna selection (TAS) scheme in the broadcast (BC) phase for achieving full diversity gain. In particular, we obtain the upper bound of BER at the multiple access (MA) phase, while deriving the exact BER performance at the BC phase. To the best our knowledge, the mathematical analysis in this paper is the first analytical results in the literature. Through computer simulations, the analytical results are validated.

Keywords: IoT networks; Physical-layer network coding (PNC); Two-way relay channel (TWRC); Bit-error rate (BER); Multiple antennas

1. Introduction

Physical-layer network coding (PNC) technique has been considered as one of the most promising solutions to provide reliability and reduce the delay in 6G wireless communication systems [1]. The PNC technique has been proposed for two-way relay channel (TWRC), which can be regarded as practical packet relaying communication scenarios in smart home and space optical communication networks [2,3]. The PNC has been known to improve twice the throughput performance compared with traditional relaying protocols as shown in Fig. 1 and can achieve higher rates than non-orthogonal multiple access (NOMA) in the TWRC [4].

Various studies to analyze performance of the PNC technique have been proposed in various antenna configurations in the literatures. The error analysis of the PNC with a single antenna at both transmitter and receiver was investigated in [5–7]. In [5], bit-error rate (BER) and sum-rate performances of the PNC were analyzed in TWRC. In [6], the exact BER performance was derived with the binary phase shift keying (BPSK) modulation over Rayleigh fading channels. In [7], a transmit power allocation strategy at both source nodes was proposed to minimizing the BER performance at the relay node.

For better reliability of the PNC technique, multiple antennas have been applied to the PNC technique. In [8], the PNC technique was applied to a multi-user massive multiple input multiple output (MIMO) system. However, MIMO techniques in general require a large number of radio frequency (RF) chain and thus induce significant signaling overhead [9]. In order to reduce such an implementation cost of the MIMO system, the antenna selection scheme has been adapted, while providing full diversity gain in TWRC [10]. In [11], an antenna subset selection technique that maximizes the minimum symbol error rate of MA and BC phases was proposed for the zero-forcing beamforming based MIMO PNC system. In [12], upper and lower bounds of BER with BPSK modulation were derived for MIMO PNC-based TWRC. With comparing the PNC with space–time block codes, it was shown that the transmit antenna selection scheme yields better performance in the TWRC. In [13], furthermore, an antenna selection scheme

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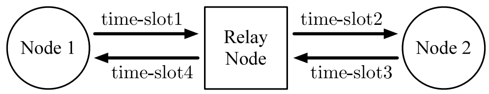
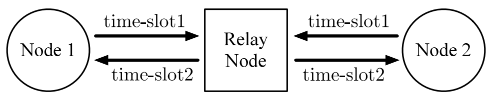


(a) Tranditional transmission scheme



(b) Physical-layer network coding scheme

Fig. 1. Transmission schemes in two-way relaying channel.

at source nodes was proposed and its error performance was analyzed especially in the MA phase. Among various schemes, transmit antenna selection (TAS) based on max–min criterion has received much attention due to its attraction to achieve full diversity order [12,14,15]. To the best of our knowledge, however, the error performance of the max–min TAS scheme has not been mathematically analyzed rigorously especially when the QPSK modulation was used.

In this paper, we mathematically analyze the BER performance of the PNC with the QPSK modulation, assuming multiple antennas at the relay node. In the (first) MA phase, the relay node detects a network-coded symbol (XOR-ed) from multiple superimposed signals from two source nodes by utilizing an optimal joint maximum likelihood (JML) detector. In the (second) BC phase, the relay node exploits the max–min TAS scheme in order to obtain the diversity gain. We analyze the BER performance of both phases, and derive the end-to-end (E2E) performance in terms of BER and diversity gain.

2. System model

We consider the TWRC consisting of two users and one relay node where the two users desire to exchange their information with each other. The relay node is equipped with N antennas, while the users have a single antenna. One cycle of PNC is divided into the MA phase and BC phase in time division manner.

2.1. Multiple access (MA) phase

In the MA phase, the user 1 and user 2 transmit QPSK modulated symbol, x_1 and x_2 respectively, to the relay node simultaneously under perfect synchronization. Then a received signal at the relay node is given by

$$\mathbf{y}_{\mathrm{r}} = \sqrt{P_1}\mathbf{h}_1 x_1 + \sqrt{P_2}\mathbf{h}_2 x_2 + \mathbf{w}_{\mathrm{r}}, \tag{1}$$

where $\mathbf{y}_{\mathrm{r}} \in \mathbb{C}^{N\times 1}$ denotes received signal vector, P_i ($i \in \{1, 2\}$) denotes the transmit power of user i, $\mathbf{h}_i \in \mathbb{C}^{N\times 1}$ denotes the channel vector from the user i to the relay node which is assumed to follow independently identically distributed (i.i.d.) complex normal Gaussian distribution, i.e., $\mathbf{h}_i \sim \mathcal{CN}(0, \mathbf{I}_N)$, and $\mathbf{w}_{\mathrm{r}} \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(0, N_0\mathbf{I}_N)$ is the additive white Gaussian noise (AWGN) vector.

At the relay node, a network-coded symbol by XOR, x_{r}, can be detected after x_1 and x_2 symbols are detected from $\mathbf{y}_{\mathrm{r}}$ by exploiting the *optimal* JML decoding as:

$$\hat{x}_{\mathrm{r}} = \hat{x}_1 \oplus \hat{x}_2,$$
$$\left(\hat{x}_1, \hat{x}_2\right) = \underset{(x_1,x_2)\in\mathcal{X}^2}{\arg\min}\|\mathbf{y}_{\mathrm{r}} - \sqrt{P_1}\mathbf{h}_1 x_1 - P_2\mathbf{h}_2 x_2\|^2, \tag{2}$$

where $\oplus$ denotes the symbol-wise XOR operator and $\mathcal{X}$ denotes the set of all candidate QPSK symbols.

2.2. Broadcast (BC) phase

The relay node broadcasts the detected network-coded symbol to the both users. We exploit the max–min TAS scheme for low complexity and achieving full diversity. A transmission antenna is selected by following:

$$t = \underset{n\in\{1,\ldots,N\}}{\arg\max}\left(\min\left(|g_{1,n}|^2, |g_{2,n}|^2\right)\right), \tag{3}$$

where t denotes the index of transmission antenna of the relay node and $g_{n,i} \sim \mathcal{CN}(0, 1)$ ($i \sim \{1, 2\}$) denotes i.i.d. channel coefficient from n-th ($n \sim \{1, 2, \ldots, N\}$) antenna of relay node to the user i.

At the user i, the received signal, y_i, is then given by

$$y_i = \sqrt{P_{\mathrm{r}}} g_{i,t} x_{\mathrm{r}} + n_i, \tag{4}$$

where P_{r} denotes the transmit power at the relay node and $n_i \sim \mathcal{CN}(0, N_0)$ denotes the AWGN at user i. A receiver of each user is equipped with ML detector and then the network-coded symbol $x_{\mathrm{r},i}$ is detected as

$$\hat{x}_{\mathrm{r},i} = \underset{x_{\mathrm{r}}\in\mathcal{X}}{\arg\min}\left|y_i - \sqrt{P_{\mathrm{r}}} g_{i,t} x_{\mathrm{r}}\right|^2. \tag{5}$$

Since each user knows its own symbol x_i transmitted in the MA phase, the user i can decode the desired symbol $x_{j,i}$ ($j \in \{1, 2\},\ j \neq i$) as follows:

$$\hat{x}_{j,i} = \hat{x}_{\mathrm{r},i} \oplus x_i, \tag{6}$$

where $\hat{x}_{j,i}$ denotes the x_j detected at the user i.

3. BER performance analysis

In this section, we mathematically analyze the upper bound of BER in the MA phase and the exact average BER in the BC phase, respectively. In addition, the diversity order of E2E of PNC with the TAS in TWRC is analyzed through analyzed BER results.

3.1. BER in the MA phase

We deal with the error case when both users transmit all zero bits due to symmetry for other bit combinations and we first focus on deriving BER of the first XOR bit.

The error cases of the first bit are when the relay node detects one of eight error constellation points which indicate the first XOR bit is 1. Then, by using the union bound on detection error probability in complex N-dimensional space, the conditional BER for given $\mathbf{h}_1$ and $\mathbf{h}_2$ is given by

$$\Pr(\mathcal{E}_b|\mathbf{h}_1, \mathbf{h}_2) \leq \sum_{m=1}^{8} Q\left(\sqrt{\frac{\delta_m^2}{2N_0}}\right), \tag{7}$$

where $\mathcal{E}_b$ denotes bit error event, $\delta_m = \left\|\sqrt{P_1}\mathbf{h}_1 a_m + \sqrt{P_2}\mathbf{h}_2 b_m\right\|$ $(m \in \{1, 2, \ldots, 8\})$, a_m and b_m are the Euclidean distances between transmit symbol and error symbol of user 1 and user 2 for mth error constellation point respectively. For notational convenience, let $Z_m \triangleq \delta_m^2/(2N_0)$. We can obtain that Z_m follows Erlang distribution with mean $N\gamma_m$ and variance $N\gamma_m^2$ where $\gamma_m = (P_1|a_m|^2 + P_2|b_m|^2)/(2N_0)$. We derive an upper bounded BER of the first XOR network-coded bit as follows [16]:

$$P_{b,\mathsf{MA}} \le \mathbb{E}\left[\sum_{m=1}^{8} Q\left(\sqrt{Z_m}\right)\right] = \sum_{m=1}^{8}\int_0^\infty Q\left(\sqrt{z}\right) f_{Z_m}(z)dz$$
$$= \sum_{m=1}^{8}\frac{1}{2}\left[1 - \sum_{k=0}^{N-1}\binom{2k}{k}\sqrt{\frac{1}{1+2/\gamma_m}}\frac{1}{(2\gamma_m+4)^k}\right]. \quad (8)$$

3.2. BER in the BC phase

Let define a link with minimum channel gain and a link with maximum channel gain for the selected transmit antenna at the relay node as *weak* link and *strong* link, respectively. Then, an average BER at each user in the BC phase is given by

$$P_{b,\mathsf{BC}} = \frac{1}{2}P_{b,\mathsf{weak}} + \frac{1}{2}P_{b,\mathsf{strong}}, \quad (9)$$

where $P_{b,\mathsf{weak}}$ and $P_{b,\mathsf{strong}}$ denote the BERs of users with the weak and strong links, respectively.

The BER of user with the weak channel gain is derived in [14] as follows

$$P_{b,\mathsf{weak}} = \frac{N}{2}\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^k}{k+1}\left\{1 - \frac{1}{\sqrt{1+\frac{4(k+1)}{\gamma}}}\right\}, \quad (10)$$

where $\gamma = P_\mathsf{r}/N_0$.

To derive the BER of user with strong link, we should obtain the p.d.f. of its channel gain. The p.d.f. of $S \triangleq \max(|g_{t,1}|^2, |g_{t,2}|^2)$ for t given in (3) can be calculated as

$$f_S(x) = 2N\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^k}{2k+1}e^{-x}\left(1 - e^{-x(2k+1)}\right). \quad (11)$$

Consequently, we exactly derive the BER of user with strong link for QPSK modulation as follows:

$$P_{b,\mathsf{strong}} = \int_0^\infty Q\left(\sqrt{\gamma x}\right) f_S(x)dx$$
$$= \int_0^\infty \left(\int_{\sqrt{\gamma x}}^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv\right) 2N\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^k}{2k+1} \times e^{-x}\left(1 - e^{-x(2k+1)}\right)dx$$
$$= N\sum_{k=0}^{N-1}\binom{N-1}{k}\frac{(-1)^k}{2k+1}\left(1 - \frac{1}{2(k+1)} - \frac{1}{\sqrt{1+\frac{2}{\gamma}}} + \frac{1}{2(k+1)}\frac{1}{\sqrt{1+\frac{4(k+1)}{\gamma}}}\right). \quad (12)$$

Finally, by substituting (10) and (12) into (9), the exact BER in the BC phase is obtained by

$$P_{b,\mathsf{BC}} = \frac{N}{2}\sum_{k=0}^{N-1}\binom{N-1}{k}(-1)^k\left[\frac{1}{2(k+1)}\left(1 - \frac{1}{\sqrt{1+\frac{4(k+1)}{\gamma}}}\right) + \frac{1}{2k+1}\left\{1 - \frac{1}{\sqrt{1+\frac{2}{\gamma}}} - \frac{1}{2(k+1)}\left(1 - \frac{1}{\sqrt{1+\frac{4(k+1)}{\gamma}}}\right)\right\}\right]. \quad (13)$$

3.3. Diversity order of PNC in TWRC

In this subsection, we further analyze the diversity order of E2E of PNC in TWRC with the relay node using TAS scheme. First, the BER of E2E of PNC in the TWRC can be given by combination of average BER performances derived in the MA and BC phases. Note that bit error event of E2E of PNC happens when bit detection error occurs only once during MA phase and BC phase. Then, the BER of E2E of PNC is given by

$$P_{b,\mathsf{E2E}} = P_{b,\mathsf{MA}}(1 - P_{b,\mathsf{BC}}) + P_{b,\mathsf{BC}}(1 - P_{b,\mathsf{MA}}). \quad (14)$$

The diversity order is in general defined as the behavior of error probability in the high SNR regime:

$$\eta = -\lim_{\gamma\to\infty}\frac{\log P_{b,\mathsf{E2E}}}{\log\gamma}, \quad (15)$$

where η denotes the diversity order and $\gamma = P/N_0$ denotes SNR. We use Taylor series expansion to (14). The first-order expansions of $P_{b,\mathsf{MA}}$ and $P_{b,\mathsf{BC}}$ are given by

$$P_{b,\mathsf{MA}} \approx \frac{3(2^{N-1})}{\sqrt{\pi N}}\gamma^{-N} + O\left(\gamma^{-N}\right), \quad (16)$$

$$P_{b,\mathsf{BC}} \approx \frac{4^{N-1}\Gamma(N+1/2)}{\sqrt{\pi}}\gamma^{-N} + O\left(\gamma^{-N}\right), \quad (17)$$

where $P_1 = P_2 = P_r = P$ and $\Gamma(\cdot)$ denotes Gamma function. Plugging (16) and (17) into (15), we can observe the diversity order of the E2E of PNC in TWRC is equal to the number of relay antenna, N, as follows:

$$\eta = -\lim_{\gamma\to\infty}\frac{\log(1-(1-P_{b,\mathsf{MA}})(1-P_{b,\mathsf{BC}}))}{\log\gamma} = N.$$

From these results, it is shown that TWRC with relay node using the max–min TAS scheme achieves full diversity order without diversity loss.

4. Simulation results

In this section, we validate the mathematically analyzed BER performances of PNC using max–min TAS in the TWRC with multiple relay antenna. For simulation results, we define the average received SNR for the MA phase, BC phase, and E2E as $\rho_{\mathsf{MA}} \triangleq \left(\mathbb{E}\left[\|\mathbf{h}_1\|^2 P_1/N_0\right] + \mathbb{E}\left[\|\mathbf{h}_2\|^2 P_2/N_0\right]\right)/2$, $\rho_{\mathsf{BC}} \triangleq \left(\mathbb{E}\left[|g_{1t}|^2 P_\mathsf{r}/N_0\right] + \mathbb{E}\left[|g_{2t}|^2 P_\mathsf{r}/N_0\right]\right)/2$, and $\rho_{\mathsf{E2E}} \triangleq (\rho_{\mathsf{MA}} + \rho_{\mathsf{BC}})/2$, respectively. We set all transmit power is the same power, i.e., $P_1 = P_2 = P_\mathsf{r}$.

In Fig. 2, the simulation results show the BER performance in the MA and BC phases when $N = 2$ and 8. The exactly

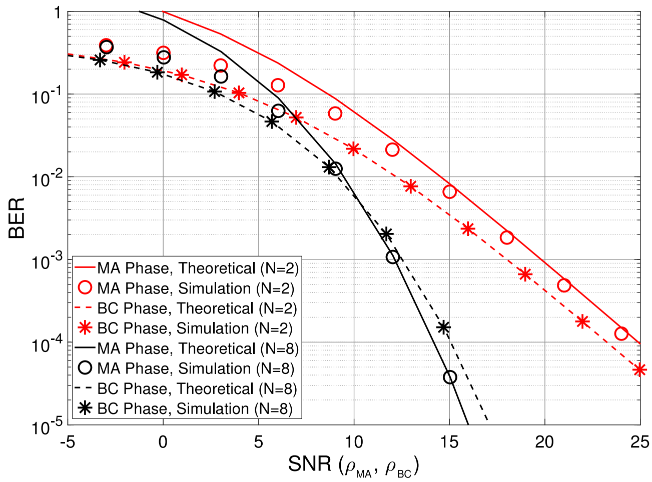


Fig. 2. BER performance of the MA phase and BC phase of PNC in TWRC for varying ρ_{MA} and ρ_{BC} when $N = 2$ and 8.

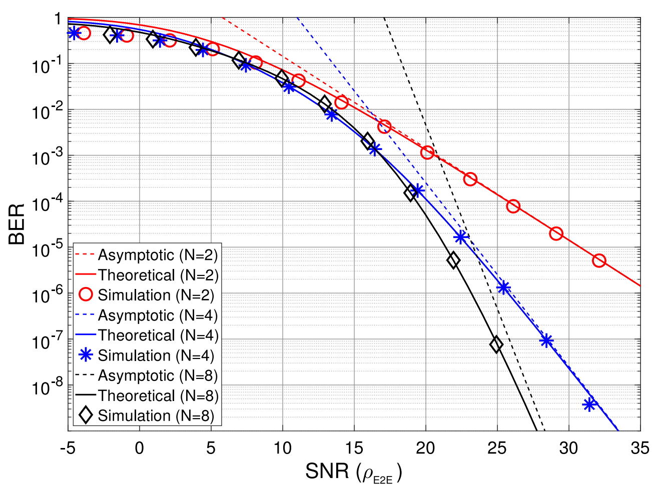


Fig. 3. BER performance of E2E of PNC in TWRC for varying ρ_{E2E} when $N = 2$, 4 and 8.

analyzed BER results of BC phase are matched well with the simulation results. The BER results of MA phase become accurate as the received SNR increases.

Fig. 3 verifies the BER performance and diversity order of E2E of PNC according to $N = 2,\ 4$ and 8. The analysis results of diversity order marked as 'asymptotic' are matched worse with lower SNR regime or a large number of relay antenna. Because we derive the diversity order by using Taylor series expansion and omits all series terms except the highest-degree term.

5. Conclusion

In this paper, we mathematically characterized bit-error rates (BERs) of physical-layer network coding in the two-way relay channel with multiple relay antenna. At multiple access (MA) phase, the relay node detects the XOR network-coded signal from superimposed signal by using joint maximum likelihood detector. For full diversity gain, max–min antenna selection scheme is adapted at broadcast (BC) phase. Analyzed performances of upper bounded BER at MA phase and exact BER at BC phase were validated by comparing with the computer simulation results. In addition, we analyzed end-to-end BER and diversity order in this system.

For a higher modulation order, our analysis approach is still available because we simply need to consider more error cases in the union upper bound technique.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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